

Warm-Up

| CST/CAHSEE: Grade 8 Algebra I 15.0 | Review: Grade 6 AF 2.3 |
|---|--|
| <p>Andy's driving speed for a 4-hour trip was 45 miles per hour. During his first 3 hours he drove 40 miles per hour. What was his average speed for the last hour of the trip?</p> <p>A) 50 miles per hour</p> <p>B) 60 miles per hour</p> <p>C) 65 miles per hour</p> <p>D) 70 miles per hour</p> | <p>If a freight train travels at a speed of 20 miles per hour for 6 hours, how far will it travel?</p> <p>•Show how to find the answer using a different approach.</p> |
| Current: Grade 7 AF 4.2 | Other: |
| <p>A train traveled at an average speed of 45 miles per hour for the first 2 hours and 30 miles per hour for the last 3 hours. What is the total number of miles that the train traveled?</p> <p>• Did the train travel farther during the first 2 hours or during the last 3 hours?</p> | <p>Solve the proportion $\frac{1}{3} = \frac{n}{15}$ three ways.</p> |

Today's Objective/Standards: Use the bar model to solve rate problems. Algebra I 15.0

Using Bar Models to Solve Rate Problems In Algebra I

Bar Models can be setup a variety of ways. **Bar Models** provide students with a way to approach word problems, help them define their variable and offer an **alternative to the distance = rate \times time formula**. Ultimately, we still want students to be able to set up the equation.

Example 1: “Average Speed” Problem

Nakia’s driving speed for a 4-hour trip was 60 miles per hour. During her first 3 hours she drove 55 miles per hour. What was her average speed for the last hour of the trip?

BAR MODEL

Let x = the average speed for the last hour
How many hours?

| 1 st hour | 2 nd hour | 3 rd hour | 4 th hour | |
|----------------------|----------------------|----------------------|----------------------|-------|
| 60 | 60 | 60 | 60 | = 240 |
| Average speed | | | | |

| 1 st hour | 2 nd hour | 3 rd hour | 4 th hour | |
|----------------------|----------------------|----------------------|----------------------|-------|
| 55 | 55 | 55 | x | = 240 |
| Actual speed | | | | |

165 miles

So, 240 miles subtracted by 165 miles
is 75 miles.

\therefore the average speed for the last hour is 75 mph

Let x = the average speed
for the last hour

$$4(60) = 3(55) + 1(x)$$

$$240 = 165 + x$$

$$240 - 165 = 165 - 165 + x$$

$$75 = x$$

\therefore the average speed for the
last hour is 75 mph

You Try 1

Phil's driving speed for a 6-hour trip was 65 miles per hour. During his first 4 hours, he drove 60 miles per hour. What was his average speed for the last 2 hours of the trip?

You Try 1

Phil's driving speed for a 6-hour trip was 65 miles per hour. During his first 4 hours, he drove 60 miles per hour. What was his average speed for the last 2 hours of the trip?

BAR MODEL

Let x = the average speed for the last 2 hours
How many hours?

| 1 st | 2 nd | 3 rd | 4 th | 5 th | 6 th | |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-------|
| 65 | 65 | 65 | 65 | 65 | 65 | = 390 |
| Average speed | | | | | | |

| 1 st | 2 nd | 3 rd | 4 th | 5 th | 6 th |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 60 | 60 | 60 | 60 | x | x |



Actual speed
240 miles

So, 390 miles subtracted by 240 miles
is 150 miles for the last 2 hours.

∴ the average speed for the last hour is 75 mph

Let x = the average speed
for the last 2 hours

$$6(65) = 4(60) + 2(x)$$

$$390 = 240 + 2x$$

$$390 - 240 = 240 - 240 + 2x$$

$$150 = 2x$$

$$\frac{150}{2} = \frac{2x}{2}$$

$$75 = x$$

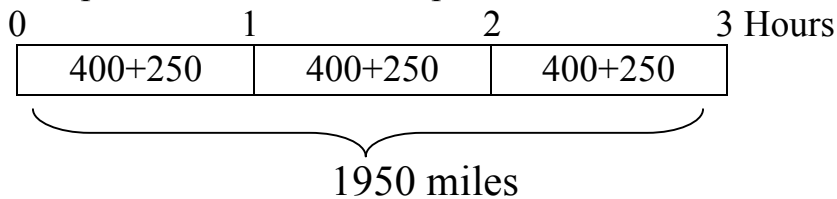
∴ the average speed for the
last 2 hours was 75 mph

Example 2: “Opposite Directions” Problem

Two airplanes left the same airport traveling in opposite directions. If one airplane averages 400 miles per hour and the other airplane averages 250 miles per hour, in how many hours will the distance between the two planes be 1625 miles?

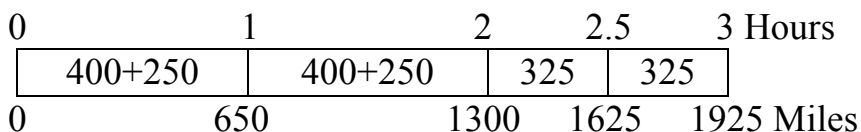
BAR MODEL

Let x = the number of hours until the planes are 1625 miles apart



(Discuss how the distance each plane travels is the total distance that they are apart.)

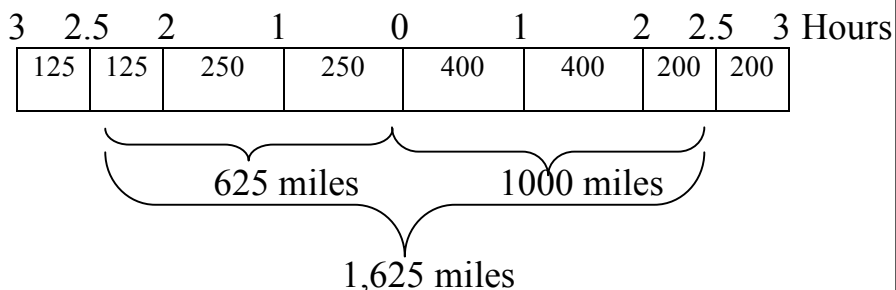
So it takes longer than 2 hours but less than 3 hours.



If we split the 3rd hour in half, we see that it will take 2 ½ hours.

∴ It will take 2 ½ hours.

OR



∴ It will take 2 ½ hours.

Let x = the number of hours until the planes are 1625 miles apart

$$400x + 250x = 1625$$

$$650x = 1625$$

$$\frac{650x}{650} = \frac{1625}{650}$$

$$x = 2.5$$

∴ It will take 2 ½ hours.

You Try 2

Two airplanes leave New York at the same time in opposite directions. If one airplane averages 300 miles per hour and the other airplane averages 200 miles per hour, in how many hours will the distance between the two planes be 2500 miles?

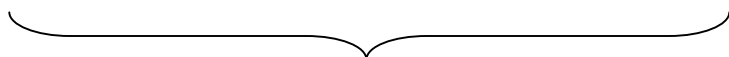
You Try 2

Two airplanes leave New York at the same time in opposite directions. If one airplane averages 300 miles per hour and the other airplane averages 200 miles per hour, in how many hours will the distance between the two planes be 2500 miles?

BAR MODEL

Let x = the number of hours until the planes are 2500 miles apart

| | | | | | |
|---------|---------|---------|---------|---------|---------|
| 0 hours | 1 | 2 | 3 | 4 | 5 |
| 200+300 | 200+300 | 200+300 | 200+300 | 200+300 | 200+300 |



2500 miles

(Discuss how the distance each plane travels is the total distance that they are apart.)

∴ It will take 5 hours.

Let x = the number of hours until the planes are 2500 miles apart

$$\begin{aligned}200x + 300x &= 2500 \\500x &= 2500 \\ \frac{500x}{500} &= \frac{2500}{500} \\x &= 5\end{aligned}$$

∴ It will take 5 hours.

Example 3: “Overtake/Catch Up” Problem

A car leaves Oakland for Las Vegas traveling an average of 40 mph. Two hours later, a truck leaves the same place in Oakland for Las Vegas at 60 mph. How long will it be before the truck overtakes the car?

(Discuss with students that the question is a little unclear. Does it mean how long from when the car left or the truck left? Usually it is asking how long after the second object left. It is a good idea to have students answer both.)

BAR MODEL

Let x = the time it takes the truck to overtake the car

Car

| | | | | | | |
|---------|----|----|-----|-----|-----|-----|
| 0 hours | 1 | 2 | 3 | 4 | 5 | 6 |
| 40 | 40 | 40 | 40 | 40 | 40 | 40 |
| 0 miles | 40 | 80 | 120 | 160 | 200 | 240 |

Truck

| | | | | | | |
|---------|---|----|----|-----|-----|-----|
| 0 hours | 0 | 0 | 1 | 2 | 3 | 4 |
| 0 | 0 | 60 | 60 | 60 | 60 | 60 |
| 0 miles | 0 | 0 | 60 | 120 | 180 | 240 |

(Discuss how the truck is at zero hours for the car’s first two hours.)

∴ The time it takes the truck to overtake the car is 4 hours.

Let x = the time it takes the truck to overtake the car

$$40(x + 2) = 60x$$

$$40x + 80 = 60x$$

$$40x - 40x + 80 = 60x - 40x$$

$$80 = 20x$$

$$\frac{80}{20} = \frac{20x}{20}$$

$$4 = x$$

∴ The time it takes the truck to overtake the car is 4 hours.

You Try 3

A sailboat leaves San Francisco for Los Angeles at 30 mph. A motorboat leaves the same place two hours later in San Francisco for Los Angeles at 50 mph. How long will it take until the motorboat catches up to the sailboat?

You Try 3

A sailboat leaves San Francisco for Los Angeles at 30 mph. A motorboat leaves the same place two hours later in San Francisco for Los Angeles at 50 mph. How long will it take until the motorboat catches up to the sailboat?

BAR MODEL

Let x = the time it takes the motorboat to overtake the sailboat

Sailboat

| | | | | | |
|---------|----|----|----|-----|-----|
| 0 hours | 1 | 2 | 3 | 4 | 5 |
| 30 | 30 | 30 | 30 | 30 | |
| 0 miles | 30 | 60 | 90 | 120 | 150 |

Motorboat

| | | | | | |
|---------|---|----|----|-----|-----|
| 0 hours | 0 | 0 | 1 | 2 | 3 |
| 0 | 0 | 50 | 50 | 50 | |
| 0 miles | 0 | 0 | 50 | 100 | 150 |

(Discuss how the motorboat is at zero hours for the sailboat's first two hours.)

∴ The time it takes the motorboat to overtake the sailboat is 3 hours.

Let x = the time it takes the motorboat to overtake the sailboat

$$30(x + 2) = 50x$$

$$30x + 60 = 50x$$

$$30x - 30x + 60 = 50x - 30x$$

$$60 = 20x$$

$$\frac{60}{20} = \frac{20x}{20}$$

$$3 = x$$

∴ The time it takes the motorboat to overtake the sailboat is 3 hours.

Example 4: “Meet” Problem

A car leaves San Francisco for Anaheim traveling an average of 65 mph. At the same time, another car leaves Anaheim for San Francisco traveling an average of 60 mph. If it is 500 miles between San Francisco and Anaheim, how long until the two cars meet?

BAR MODEL

Let x = the time until the two cars meet

| | | | | | |
|---------|---------|---------|---------|-----|-------------|
| 0 | 1 | 2 | 3 | 4 | Hours |
| 65 + 60 | 65 + 60 | 65 + 60 | 65 + 60 | | |
| 0 | 125 | 250 | 375 | 500 | Total miles |

(Discuss how the total distance is how much closer the cars are to each other.)

∴ It will take the cars 4 hours to meet.

Let x = the time until the two cars meet

$$65x + 60x = 500$$

$$125x = 500$$

$$\frac{125x}{125} = \frac{500}{125}$$

$$x = 4$$

∴ It will take the cars 4 hours to meet.

You Try 4

Anahi leaves her house for her friend's house walking at 4 mph. At the same time, her friend Jessica leaves her house for Anahi's house walking at 3 mph. If they live 21 miles apart, how long will it take until Anahi and Jessica meet?

You Try 4

Anahi leaves her house for her friend's house walking at 4 mph. At the same time, her friend Jessica leaves her house for Anahi's house walking at 3 mph. If they live 21 miles apart, how long will it take until Anahi and Jessica meet?

BAR MODEL

Let x = the time until the two friends meet

| | | | | |
|-------|---|-------|----|-------------|
| 0 | 1 | 2 | 3 | Hours |
| 3 + 4 | | 3 + 4 | | 3 + 4 |
| 0 | 7 | 14 | 21 | Total miles |

(Discuss how the total distance is how much closer the friends are to each other.)

∴ It will take them 3 hours to meet.

Let x = the time until the two friends meet

$$4x + 3x = 21$$

$$\begin{aligned} 7x &= 21 \\ \frac{7x}{7} &= \frac{21}{7} \\ x &= 3 \end{aligned}$$

∴ It will take them 3 hours to meet.